A Quantum Computing-based System for Portfolio Optimization using Future Asset Values and Automatic Reduction of the Investment Universe

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Abstract. One of the problems in quantitative finance that has received the greatest attention is the portfolio optimization problem. Regarding its solving, this problem has been approached using different techniques, with those related to quantum computing being especially prolific in recent years. In this study, we present a system called *Quantum Computing-based System for Portfolio Optimization with Future Asset Values and Automatic Universe Reduction* (`Q4FuturePOP`), which deals with the Portfolio Optimization Problem considering the following innovations: *i*) the developed tool is modeled for working with future prediction of assets, instead of historical values; and *ii*) `Q4FuturePOP` includes an *automatic universe reduction* module, which is conceived to intelligently reduce the complexity of the problem. We also introduce a brief discussion about the preliminary performance of the different modules that compose the prototypical version of `Q4FuturePOP` developed.

Keywords: Quantum Computing, Portfolio Optimization, Quantum Annealer, D-Wave, Optimization

1 Introduction

The present work aims to describe a quantum computing (*QC*, [1]) based system for solving the portfolio optimization problem (*POP*, [2]). Briefly explained, the *POP* intends to find the optimum asset allocation with the objective of *i*) maximizing the expected return and *ii*) minimizing the financial risk. More specifically, and following the Markowitz *POP* formulation, the financial risk is calculated based on the diversification of the built portfolio. Following this philosophy, the system tends to distribute the whole budget into different and uncorrelated assets rather than investing large amounts of money into the highest expected, albeit correlated, returns.

Formally described, the problem to be solved counts with *i*) a group of N assets $\mathcal{A} = \{a_0, \ldots, a_i, \ldots, a_{N-1}\}$; *ii*) a dataset $AD = \{ad_0, \ldots, ad_i, \ldots, ad_{N-1}\}$, in which ad_i is a list of historical values $ad_{i,k}$ of an asset a_i, representing k a specific day within the complete period of K days, and *iii*) a total bd budget.

Thus, the objective is to find the most promising assets in which to invest this budget, considering that i) all bd must be invested and ii) the proportion of bd that can be allocated to each asset is conditioned by the variable p. More concretely, and considering w_i as the proportion of bd invested on the asset a_i, this w_i can be represented as the summation of any proportions p_i in $P = \{p_0 = bd, p_1 = bd/2, ...p_{p-1} = bd/2^{p-1}, 0\}$, Furthermore, it should be deemed that $w_i <= bd$. With this notation in mind, the goal is to find the $W = \{w_0, w_1, ..., w_{N-1}\}$ that maximizes the total expected return while minimizing the financial risk.

The system described in this paper, coined *Quantum Computing-based System for Portfolio Optimization with Future Asset values and automatic universe reduction* (`Q4FuturePOP`), is a QC-based scheme for dealing with the POP considering the following innovations:

- *Future projected values*: most of the QC based techniques proposed in the literature solve the POP using as input historical dataset values for $\mathcal{A}$ [3]. In other words, developed solvers select the appropriate W considering the past values of the group of available assets. On the contrary, for `Q4FuturePOP`, the input dataset is composed of future predictions to try to account for realistic environments in the POP formulation. Using this input, `Q4FuturePOP` builds the complete dataset used for formulating the POP problem considering future projected values of $\mathcal{A}$. Thus, the W chosen by the system is based on future predictions instead of historical values.
- *Automatic universe reduction*: in order to calculate W in a more efficient way, a search space reduction mechanism has been implemented in `Q4FuturePOP`. This mechanism works as follows: first, `Q4FuturePOP` takes as input the complete universe of $\mathcal{A}$. With this set of assets, the system conducts a number E of preliminary executions which are used by `Q4FuturePOP` for detecting a subgroup of promising assets. After these preliminary executions, `Q4FuturePOP` builds an alternative $\mathcal{A}'$, which is a subgroup of $\mathcal{A}$ ($\mathcal{A}' \subseteq \mathcal{A}$). Using this newly generated $\mathcal{A}'$, the problem is finally executed, and the obtained outcomes are returned to the user. Thanks to this procedure, the complexity of the problem to solve is automatically decreased, allowing the system to reach a higher level of accuracy.

The rest of the paper is structured as follows. Section 2 presents a brief overview of the background related to QC and POP. In Section 3, the inputs and outputs of `Q4FuturePOP` are described for the sake of understandability. After that, in Section 4, the whole system is described. Then, in Section 5, we discuss the preliminary performance of `Q4FuturePOP`. Section 6 finishes this work by outlining some of the planned future work.

2 Related Work

The first POP-focused paper including real quantum experiments was published in 2015, in which the authors solved the problem using a prototype of the D-Wave's quantum annealer [4]. With only 512 qubits at their disposal, the authors worked with a pool of 15 assets in which to invest. A second study focused on this topic was presented in 2017, exploring a specific investment case related to the Abu Dhabi Securities

Exchange [5]. In the following years, some interesting theoretical papers appeared, exploring different formulations and their possible resolutions using quantum approaches. Examples of this scientific trend can be found in [6] and [7]. These papers, published in 2018 and 2019, respectively, theorize the implementation of two algorithms without actually testing them on real quantum devices.

Also in 2019, some advanced approaches were presented. In [8], for example, the authors employ a hybrid algorithm in which the quantum module is executed by the D-Wave 2000Q computer, making use of the *reverse annealing* mechanism to improve the solutions found by a classical greedy algorithm. In the same year, the first paper focused on gate-based quantum computers was published, in which the authors present a *Quantum Approximate Optimization Algorithm* [9].

Particularly interesting are the papers [10] and [11], published by *Chicago Quantum* in 2020. In the first of these papers, the authors demonstrated how a hybrid solver can promisingly solve portfolios of up to 33 assets. In [11] a similar approach is proposed, where using another hybrid technique, the number of assets considered rises to 60. Finally, in [12], the same authors managed to solve problems with a size of 134 assets, making use of the D-Wave Advantage System, composed of 5436 qubits.

Since 2021, the study of the POP through the quantum paradigm has experienced a significant increase. In [13], for example, a variant of the problem known as *minimum holding time* is solved by the D-Wave quantum annealer, considering a universe of 50 assets in which to invest. In [14], the authors present a problem in which different investment bands are deemed, allowing the fixing of a maximum permissible risk. Furthermore, a quantum-gate-based method was presented in [15], where a hybrid algorithm called *NISQ-HHL* is proposed.

The study presented in [16] is especially interesting for this paper. That work consisted of a detailed analysis of the parameterization of the D-Wave's annealer. To do so, the authors use POP as a benchmarking problem, presenting a formulation of the problem that allows an investment granularity adapted to the user's needs. This study has proved to be interesting from a mathematical point of view since the formulation employed in this study is the one embraced in our work. Finally, it is worth highlighting the study presented in [17], where different quantum solvers are proposed based on both the quantum gate paradigm and the annealer. The authors of that work carried out different tests in a dynamic environment, solving problems with up to 52 assets.

As can be seen, the research conducted in recent years has been prolific. This section has attempted to briefly outline this vibrant activity. Being aware that the full state of the art is much broader, we refer interested readers to works as [18].

3 Inputs and Outputs of `Q4FuturePOP`

First, let's define some notation and terms in order to properly understand how `Q4FuturePOP` operates. From now on, we use the superscripts ^{h} for the historical data and ^{f} for the predicted future data. Furthermore, the daily return of an asset a_i at day k is $er^h_{k,i} = (ad^h_{k,i} - ad^h_{k-1,i})/ad^h_{k-1,i}$, resulting in $er^h \in \mathbb{R}^{K \times N}$ as the matrix of all the daily returns of $\mathcal{A}$ in AD^h.

Thus, Q4FuturePOP receives as inputs *i*) $AD^h \in \mathbb{R}^{K \times N}$, which is the matrix containing all the historical values of $\mathcal{A}$ assets during K days, *ii*) a list $V = \{v_0, v_1, ..., v_{N-1}\}$, in which v_i is the initial value of a_i at the time Q4FuturePOP is executed (in most cases, $v_i = a^h_{K-1,i}$); and *iii*) $Er^f = \{Er^f_0, Er^f_1, ..., Er^f_{N-1}\} \in \mathbb{R}^N$, which is the list of predicted expected returnsIt should be highlighted that these Er^f values are given by an expert from the Spanish company Welzia Management[4].

Once the input is received, the module named *Predicted Dataset Generation* (PDG) builds the complete dataset $AD^f \in \mathbb{R}^{L \times N}$, which is composed of all the projected future daily values of the whole $\mathcal{A}$ in the period that goes from the moment in which the system is executed and the following L days. All values in the generated AD^f must meet two requirements: *i*) $Cov(er^f) = Cov(er^h)$, meaning that the covariance of the input daily returns is the same as that of the daily returns generated by PDG; and *ii*) the expected return of the prices build must be the same as Er^f. How AD^f is generated is described in Section 4.1.

As output, Q4FuturePOP returns *i*) the above-mentioned W, containing the list of investment weights w_i given to each a_i; *ii*) the expected return of the chosen portfolio $er_{portfolio}$; *iii*) the risk associated with this portfolio $\sigma_{portfolio}$; and *iv*) the $SHARPE$ value. It should be considered that for the computation of outcomes *ii*, *iii* and *iv*, the Markowitz formulation of the POP has been used as a base.

4 Q4FuturePOP: Quantum Computing based System for Portfolio Optimization with Future Asset values and automatic universe reduction

Q4FuturePOP consists of three interconnected modules: PDG, Assets Universe Reduction Module (AUR) and the Quantum Computing Solver Module (QCS). A schematic description of Q4FuturePOP is depicted in Figure 1, in which the relationship of the three modules is represented.

In a nutshell, PDG is devoted to generating the complete dataset of future predicted values of $\mathcal{A}$. The second module, AUR, is in charge of intelligently reducing the complete $\mathcal{A}$ into $\mathcal{A}'$, while QCS is the module that solves the POP and provides W both for AUR or for the final user (as seen in Figure 1). In the following subsections, we describe PDG, AUR and QCS in detail.

4.1 Predicted Dataset Generation Module - PDG

To simulate future prices and eventually build AD^f, the PDG starts by generating a matrix $X \in \mathbb{R}^{L-1 \times N}$ composed of random values drawn from a standard normal distribution, finding a matrix $A \in \mathbb{R}^{L-1 \times L-1}$ and a bias $b \in \mathbb{R}^{L-1 \times N}$ such that $er^f = AX + b$. For finding both A and b, PDG follows two different procedures:

- *Finding* A. The Cholesky decomposition of the covariance matrix $Cov(X)$ and $Cov(er^f)$ stands that there exist, respectively, two unique lower triangular matrix

[4] https://wz.welzia.com/

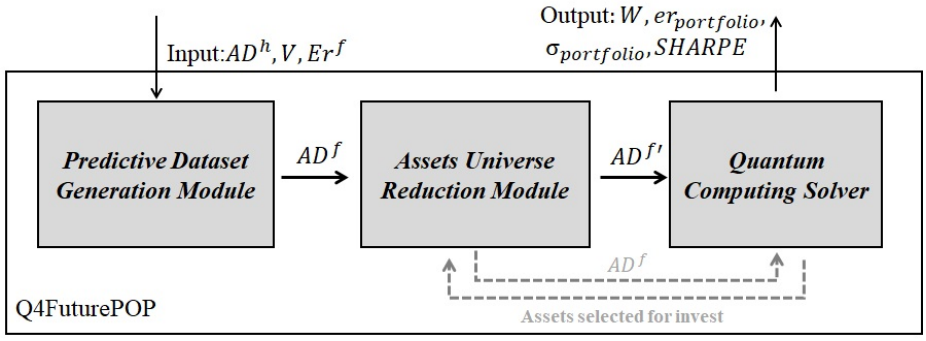

Fig. 1: A schematic description of the proposed `Q4FuturePOP`.

L_x and L_h such that $Cov(X) = L_x L_x^T$ and $Cov(er^f) = L_h L_h^T$. Considering that X is an invertible matrix, which is highly probable since the random vectors that make up X are independent by construction, we set $A^T = L_h L_x^{-1}$. Thanks to this procedure, we meet our first constraint, as $Cov(er^f) = Cov(er^h)$.

– *Finding* b. Let us consider $Y = AX$ and express the expected return as a function of the daily return. For an asset a_i we have $ln(1 + Er_i^f) = \sum_{k=1}^{L-1} ln(1 + y_{k,i} + b_i)$ if and only if $\forall k, i \subseteq \mathbb{R}^{L-1 \times N} y_{k,i} + b_i > -1$. Then, we use the Taylor-Young expansion of ln to find an approximation of $ln(1 + Er_i^f)$ as a polynomial $P_i^n(x)$. If $\forall k \subseteq \mathbb{R}^{L-1}, |y_{k,i} + x| < 1$ then $\lim_{n \to +\infty} P_i^n(x) = ln(1 + Er_i^f)$. Now, we take b_i as a real root of the polynomial $P_i^n(x) - ln(1 + Er_i^f)$ that respects the preceding constraint. If this root does not exist, the `PDG` cannot find a good solution. So, the `PDG` repeats this for all the assets in order to obtain b.

After these values are calculated following this procedure, AD^f is reconstructed from the initial values V to the predicted er^f.

4.2 Quantum Computing Solver Module - `QCS`

Despite being the last module called along the workflow of `Q4FuturePOP`, it is appropriate to describe `QCS` here since it is employed also as part of the calculation made within `AUR`. In a nutshell, the `QCS` is the module in charge of taking a complete dataset of assets containing their daily values along the considered time period and solving the POP, deciding the proportion w_i of the budget bd allocated to each a_i. We represent in Figure 2 a schematic description of `QCS`. As can be observed in that figure, this module is composed of three different components:

QUBO Builder: the POP dealt by `Q4FuturePOP` is tackled by a QC device. More specifically, the system is built to solve the problem by means of a quantum annealer. This specific type of device natively solves QUBO problems. For this reason, the first component of `QCS` has the objective of gathering the dataset provided as input and

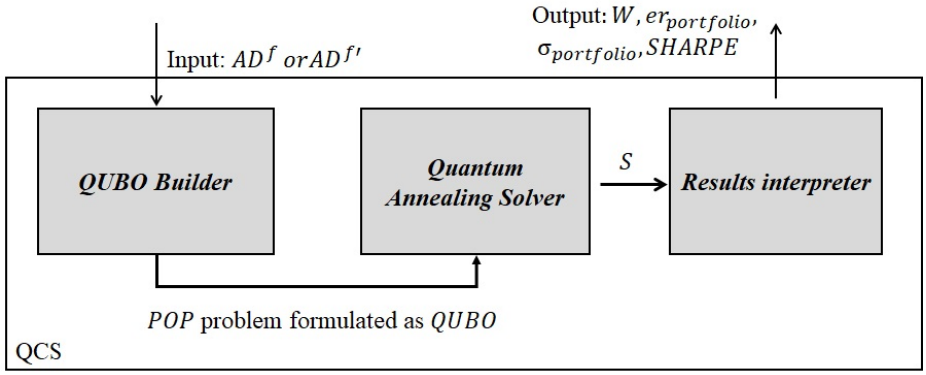

Fig. 2: A schematic description of QCS.

modeling the POP problem correctly. More specifically, for building the corresponding QUBO, we define the following Hamiltonian based on the formulation described in [16]:

$$\mathbf{H} = \alpha\mathbf{H_A} + \beta\mathbf{H_B} + \gamma\mathbf{H_C}, \tag{1}$$

where

$$\mathbf{H_A} = \sum_{i}^{N-1} w_i er_i \tag{2}$$

$$\mathbf{H_B} = -\sum_{i,j}^{N-1} Cov_{i,j} x_i x_j. \tag{3}$$

$$\mathbf{H_C} = -(\sum_{i}^{n} w_i x_i - bd)^2. \tag{4}$$

considering that er_i represents the expected return for an asset a_i, and x_i is a binary variable that is 1 if the asset a_i has a $w_i > 0$. Furthermore, the values α, β and γ are float multipliers employed to weight each term.

Quantum annealing solver: this is the central component of QCS, and where the call to the quantum annealing solver is made. As depicted in Figure 2, QCS receives as input the problem modeled as a QUBO. This problem is tackled by a quantum annealer device, such as the ones provided by DWAVE: `Advantadge_System6.1` or `Advantadge2_prototype1.1`. Quantum-inspired alternatives, such as the Fujitsu Digital Annealer, are also eligible for being part of QCS as QUBO solver. Also, hybrid approaches such as Leap's hybrid Binary Quadratic Model Solver of D-Wave can be embraced in this module. This component returns a list S of binary values, which represents the solution to the QUBO ingested by the QC device.

Results Interpreter: the solution S provided by the quantum device is not directly interpretable for the final user. For this reason, this last component oversees the obtaining of the string of binary values provided by the quantum annealing solver and

calculates the variables that will be returned as outcomes: W, $er_{portfolio}$, $\sigma_{portfolio}$ and $SHARPE$.

Thus, the QCS module is called in two different phases in the complete workflow of Q4FuturePOP. On the one hand, the QCS is called by AUR module in the process of asset universe reduction. In this iterative procedure, the QCS is executed E different times, using as input the complete dataset AD^f. Through these repetitive runs, AUR aims to detect the most interesting assets for conducting a search space reduction of the POP (more details in Section 4.3). On the other hand, the QCS is called in the last stage of the complete Q4FuturePOP execution, using as input the reduced $AD^{f'}$, and with the goal of obtaining the final W, $er_{portfolio}$, $\sigma_{portfolio}$ and $SHARPE$.

4.3 Assets Universe Reduction Module - AUR

The motivation behind the implementation of the AUR module is to face the limitations of current quantum annealers. QC is still in the early stages of development despite all the research and developments made in the field. Current QC computers suffer from limitations such as a finite number of qubits or noisy processes that impact the performance of quantum solvers [19]. For this reason, the actual stage of QC field is known as NISQ era [20].

Under this rationale, the main objective of AUR module is to decrease the complexity of the problem at hand by building a reduced sub-instance of the POP problem. Thus, with a smaller solution space, the system is able to reach higher-quality and more robust solutions. The AUR works as follows:

1. The AUR receives as input the complete set of data AD^f generated by the previously described PDG module.
2. AUR solves the POP problem using the QCS module (detailed in Section 4.2) and AD^f as input. Despite QCS provides more information, it just stores the identifiers of the assets a_i to which any proportion of bd has been allocated. Thus, AUR generates a list $\mathcal{A}'$ which is a subgroup of $\mathcal{A}$ ($\mathcal{A}' \subseteq \mathcal{A}$).
3. Until the number of execution conducted by AUR is less than E, step 2 is repeated.
4. Once the process is finished, AUR builds a reduction of AD considering the information of all the assets in $\mathcal{A}'$, and discarding all the data related to assets that have not been deemed in any of the E executions conducted by step 2.

Lastly, for helping the understanding of AUR module, we depict a schematic description of AUR in Figure 3, representing also the relation with QCS module.

5 Discussion on the preliminary performance

At this moment, Q4FuturePOP is in a prototypical stage of development, waiting to be completely validated as a whole system. Anyway, each module has been checked separately. On the one hand, the PDG has been successfully tested using a pool of predicted values provided by Welzia Management. In any case, due to extension constraints and the fact that this paper is more focused on QC, we will deepen the validation of the PDG module in a future research paper.

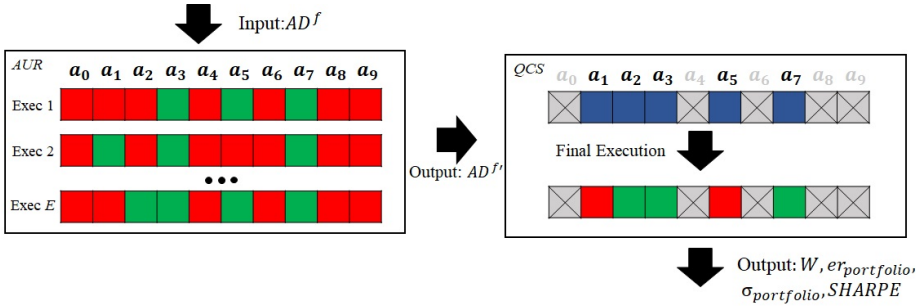

Fig. 3: A schematic description of AUR and its connection with QCS. ■ = asset with no budget allocated; ■ = asset with a budget $w_i > 0$ allocated; ■ = asset eligible for allocation in the reduced universe; ■ = asset discarded in the universe reduction process

On the other hand, AUR and QCS modules have been jointly checked using a dataset also provided by Welzia Management. This dataset contains the daily values of 53 different assets over 12 years (from 01/01/2010 to 13/12/2022). Welzia Management also provided us with a set of historical portfolios chosen by the company's experts. Thus, using the dataset as input and the historical portfolios as baseline, six different use cases have been built for validation. Each of these instances consists of an excerpt of the complete dataset, with a depth ranging from 12 to 28 months. Therefore, for each use case, AUR+QCS modules of Q4FuturePOP have been run 6 independent times, and the results provided have been compared with the portfolios built by the experts. Also, it should be noted here that the quantum solver used for the conducted tests is the Advantadge_System6.1 of D-Wave, comprised of 5610 qubits and 40134 couplers spread over a Pegasus topology.

The results of this preliminary experimentation are depicted in Table 1, in which we represent the time frame that compose each dataset, the number of available assets, and the outcomes provided by both the experts and AUR+QCS. Each instances is coined as UCX_Y_Z, where X is the ID of the dataset, Y the time frame measured in months, and Z the amount of assets deemed. Finally, it should be noted that both the employed datasets as well as the complete set of results obtained by AUR+QCS are available upon reasonable request.

Analyzing the results obtained by AUR+QCS modules, it should be noted that they have proved to be promising. These results have been approved by the experts from Welzia Management after several technical meetings. These meetings, and the fact of having the help of these experts, have been a really enriching point in the development of Q4FuturePOP. This is so since quantum-based solutions are usually analyzed from a purely academic point of view. That is, the solutions provided by the systems are usually analyzed based solely on their $SHARPE$ ratio. Although this ratio is a good indication of the quality of a solution, it does not represent the reality of the industry. High $SHARPE$ ratios can lead to triumphalist conclusions, which eventually clash with the reality of an industry with volatility that is difficult to perceive by a computer

Table 1: Results obtained from the preliminary experimentation carried out. Outcomes depicted for AUR + QCS are calculated using the averages obtained in the six independent runs. er = expected return. σ = risk.

Instance	Time Frame	Assets	Expert Results		AUR+QCS Results	
			er	σ	er	σ
UC1_12_45	31/01/2018 - 31/01/2019	45	22.56%	**2.95**	**25.89%**	5.85
UC2_12_43	05/05/2017 - 05/05/2018	43	12.95%	**3.76**	**19,30%**	5.51
UC3_24_38	01/01/2018 - 01/01/2020	35	14.05%	**4.08**	**17,63%**	6.01
UC4_28_38	24/01/2017 - 24/05/2019	38	**11.64%**	8.99	8.62%	**5.96**
UC5_15_40	01/05/2018 - 01/08/2019	40	**15.06%**	6.53	14.50%	**4.12**
UC6_20_53	09/04/2021 - 09/12/2022	53	2.81%	12,31	**6.37%**	**11.62**

(whether quantum or classical). This is why an expert's judgment when generating a portfolio is an absolutely necessary factor.

6 Conclusions and Future Work

In this paper, a quantum-based approach for solving the well-known Portfolio Optimization Problem has been presented, coined as *Quantum Computing-based System for Portfolio Optimization with Future Asset Values and Automatic Universe Reduction*. Two are the main innovations inherent to the system proposed: *i*) Q4FuturePOP is modeled for working with future prediction of assets instead of historical values; and *ii*) Q4FuturePOP includes an *automatic universe reduction* module, which is conceived to intelligently reduce the complexity of the problem. Along with the description of the system, we have also introduced a brief discussion about the preliminary performance of the different modules that compose the prototypical version of the tool.

Several research lines stem directly from the findings reported in this work. The first, and most obvious, is the validation of the complete system. Other future work includes the fine-tuning of the parameters that involve the generation of the POP's QUBO. Also, other quantum-based solvers apart from the ones provided by D-Wave are planned to be tested.

Acknowledgments

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